NCE-003-016203

NCE-003-016203 Seat No. ______

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2017

Mathematics - 2003

(Topology - II)

(Old Course)

Faculty Code : 003

Subject Code : 016203

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions in this paper.

(2) Each question carries 14 marks.

(3) All questions are compulsory.

1 Fill in the blanks : (Each question carries two marks)

(a) A space X is ________ if every open cover of X has finite sub cover.

(b) Every closed subspace of a compact space is ______.

(c) The one point compactification of a locally compact, non - compact Hausdorff space is ________ and ________.

(d) The subspace $\mathbb{R}\setminus\mathbb{Q}$ of irrationals is not connected because $\mathbb{R}\setminus\mathbb{Q}$ is not _______.

(e) If X and Y are not compact then $X \times Y$ is _______.

(f) $[0,1]\times[0,1]$ with dictionary order topology is connected but not _________.

(g) A component of a space X is a maximal ______ subset of X.

2 Attempt any **two** of the following :

(a) Prove that $X \times Y$ is locally compact if and only if both X and Y are locally compact. **7**

(b) Prove that **7**

(i) Every compact space is limit point compact.

(ii) Every compact Hausdorff space is normal.

(c) Prove that **7**

(i) Open continuous image of locally compact space is locally compact.

(ii) Any infinite set with co finite topology is compact.

3 All are compulsory :

(a) Define a filter on a set X. If X is a space, $E \subset X$ and $x \in X$ then prove that $x \in \overline{E}$ if and only if there is a filter on X which contains E and converges to x. **6**

(b) Prove that if a filter $\mathcal{F}$ converges to p then p is a cluster point of $\mathcal{F}$. **4**

(c) Let X be a space and $x \in X$. Prove that the collection of all neighbourhoods of x is a filter on X. **4**

OR

3 All are compulsory :

(a) Prove that a locally compact Hausdorff space is regular. **7**

(b) Give an example of a space which is not locally compact. **4**

(c) Prove that an infinite set with discrete topology is not compact. **3**

4 Attempt any two of the following :

(a) Suppose $f : X \to Y$ is continuous and onto. Prove that if X is connected then Y is also connected. **7**

(b) Define a path connected space. Prove that $\mathbb{R}^n$ is a path connected space. **7**

(c) Prove that a component of a space X is **7**

(i) connected

(ii) maximal connected

(iii) closed

5 Do as directed : (Each question carries two marks)

(a) Give an open cover of the subspace $\mathbb{Q}$ of rational numbers (with usual topology) which has no finite sub cover.

(b) Give the two subsets of $\mathbb{R}$ (the set of real numbers with standard topology) such that one is closed but not bounded and the other is bounded but not closed.

(c) Let $A = [0, 1)$. Is A a compact subset of $\mathbb{R}$? Give reasons for your answer.

(d) Give an example of a uncountable disconnected space.

(e) Give an example of a non compact, locally compact, Hausdorff space which is uncountable.

(f) Give an example of a finite connected space.

(g) Give the definition of a separation of a space X.